Modeling and Simulation of the Thermo-Acousto-Elastic Waves in Solids of Complex Rheology

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Two-components material point

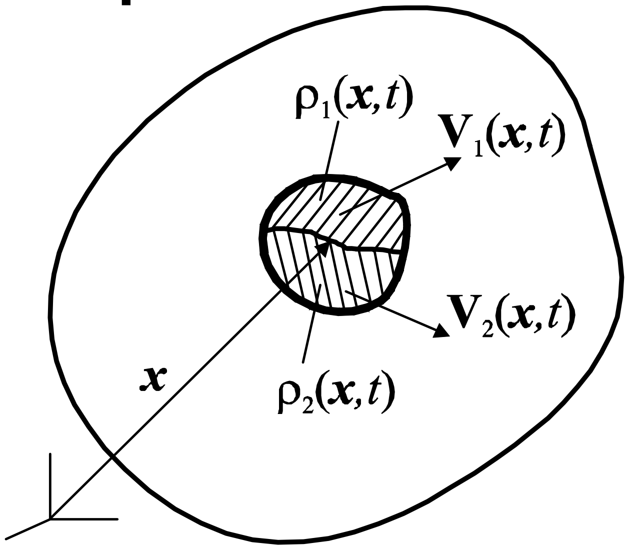

Fig.1

$$\rho_m(\mathbf{x},t) = \rho_1(\mathbf{x},t) + \rho_2(\mathbf{x},t)$$

$$\rho_m(\mathbf{x},t)\mathbf{V}_m(\mathbf{x},t) = \rho_1(\mathbf{x},t)\mathbf{V}_1(\mathbf{x},t) + \rho_2(\mathbf{x},t)\mathbf{V}_2(\mathbf{x},t)$$

Intercomponental interaction $-\mathbf{R}$, heat exchange - $\kappa(T_1 - T_2)$

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Mass conservation law

For medium

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{V}_m) = 0$$

For each component

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_1 \mathbf{V}_1) = \chi$$

$$\frac{\partial \rho_2}{\partial t} + \nabla \cdot (\rho_2 \mathbf{V}_2) = -\chi$$

χ - the source of mass

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Momentum conservation law

For medium

$$\nabla \cdot \boldsymbol{\tau}_m + \rho_m \mathbf{F}_m = \rho_1 \frac{\mathrm{d}\mathbf{V}_1}{\mathrm{d}t} + \rho_2 \frac{\mathrm{d}\mathbf{V}_2}{\mathrm{d}t} + \chi \left(\mathbf{V}_1 - \mathbf{V}_2 \right)$$

For each component

$$\boldsymbol{\nabla} \cdot \boldsymbol{\tau}_1 + \rho_1 \mathbf{F}_1 + \mathbf{Q} = \rho_1 \frac{\mathrm{d}\mathbf{V}_1}{\mathrm{d}t} + \chi \mathbf{V}_1$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{\tau}_2 + \rho_2 \mathbf{F}_2 - \mathbf{Q} = \rho_2 \frac{\mathrm{d}\mathbf{V}_2}{\mathrm{d}t} - \chi \mathbf{V}_2$$

Total stress and external forces

$$\boldsymbol{\tau}_m = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2, \quad \boldsymbol{\tau}_1 = \boldsymbol{\tau}_1(\varepsilon_1, \ldots), \ \boldsymbol{\tau}_2 = \boldsymbol{\tau}_2(\varepsilon_2, \ldots)$$

$$\rho_m \mathbf{F}_m = \rho_1 \mathbf{F}_1 + \rho_2 \mathbf{F}_2$$

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The equations of the balance of energies

$$\rho_1\left(\frac{d_1U_1}{dt}+\mathbf{V}_1\cdot\nabla U_1\right)=\tau_1\cdot\cdot\nabla\mathbf{V}_1+\frac{1}{2}\mathbf{Q}\cdot(\mathbf{V}_2-\mathbf{V}_1)-$$

$$-\nabla\cdot\mathbf{h}_1+\rho_1q_1+\chi_{m1}\left(\frac{1}{2}\mathbf{V}_1\cdot\mathbf{V}_1-U_1\right)-Q$$

$$\rho_2\left(\frac{d_2U_2}{dt}+\mathbf{V}_2\cdot\nabla U_2\right)=\tau_2\cdot\cdot\nabla\mathbf{V}_2+\frac{1}{2}\mathbf{Q}\cdot(\mathbf{V}_2-\mathbf{V}_1)-$$

$$-\nabla\cdot\mathbf{h}_2+\rho_2q_2+\chi_{m2}\left(\frac{1}{2}\mathbf{V}_2\cdot\mathbf{V}_2-U_2\right)+Q$$

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The second law of thermodynamics

$$\frac{d}{dt}\int_{(V)} \rho_1 S_1 dV \geq \int_{(V)} \left[\frac{\rho_1 q_1}{\theta_1} + \frac{Q}{\theta_2}\right] dV - \int_{(S)} \mathbf{n} \cdot \left[\frac{\mathbf{h}_1}{\theta_1} - \rho_1 \mathbf{V}_1 S_1\right] dS$$

$$\frac{d}{dt}\int_{(V)} \rho_2 S_2 dV \geq \int_{(V)} \left[\frac{\rho_2 q_2}{\theta_2} - \frac{Q}{\theta_1}\right] dV - \int_{(V)} \mathbf{n} \cdot \left[\frac{\mathbf{h}_2}{\theta_2} - \rho_2 \mathbf{V}_2 S_2\right] dS$$

Models of materials

The particles balance equations

$$\begin{cases} \dfrac{\partial \tilde{n}_1}{\partial t} + \nabla \cdot (n_1 \, \mathbf{v}_1) = J_{12} \\ \dfrac{\partial \tilde{n}_2}{\partial t} + \nabla \cdot (n_2 \, \mathbf{v}_2) = J_{21} \end{cases}$$

$$\rho_1 = m_1 n_1, \quad \rho_2 = m_2 n_2$$

$$\Longrightarrow$$

$$\rho_m = \rho_1 + \rho_2,$$

$$\rho_m \mathbf{v}_m = \rho_1 \mathbf{v}_1 + \rho_2 \mathbf{v}_2$$

Mass balance

$$\begin{cases} \dfrac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_1 \, \mathbf{v}_1) = \chi_{12} \\ \dfrac{\partial \rho_2}{\partial t} + \nabla \cdot (\rho_2 \, \mathbf{v}_2) = \chi_{21} \end{cases}$$

$$\rho_2 \ll \rho_1$$

$$\chi_{12} = -\chi_{21} = -a\tilde{\rho}_1 + b\tilde{\rho}_2$$

$$\Longrightarrow \begin{cases} \dot{\tilde{\rho}}_1 + \rho_{01}\dot{\varepsilon}_1 = -a\tilde{\rho}_1 + b\tilde{\rho}_2 \\ \dot{\rho}_2 = a\tilde{\rho}_1 - b\tilde{\rho}_2 \end{cases}$$

$$\Downarrow$$

$$\dot{\sigma}_1 = -\frac{k}{\rho_{01}} \dot{\tilde{\rho}}_1 \Longrightarrow \boxed{\dot{\sigma} + (a+b)\sigma = k\left(\rho_{01}\dot{\varepsilon}_1 + b\varepsilon_1\right)}$$

The Kelvin model

Influence of source terms on stress - strain state by chemical reaction

Let the medium B with the density ρ_{B_0} after a chemical reaction go into the medium A with the density ρ_{A0}.

Suppose that at the first approximation the chemical reactions do not depend on the stress state. That's why we can assume that sources χ_{BA} and χ_{AB} in the balance equations are known.

$$\begin{cases} \dfrac{\partial \rho_B}{\partial t} + \nabla \cdot (\rho_B \boldsymbol{v}_B) = -\dot{\chi}_{BA} \rho_{B_0} \\ \dfrac{\partial \rho_A}{\partial t} + \nabla \cdot (\rho_A \boldsymbol{v}_A) = \dot{\chi}_{AB} \rho_{A_0} \end{cases}$$

$$\chi_{BA} \rho_{B0} = \chi_{AB} \rho_{A0}$$

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For the one-dimensional case

$$\begin{cases} \dfrac{\partial \rho_B}{\partial t} + \dfrac{\partial}{\partial x}(\rho_B \boldsymbol{v}_B) = -\dot{\chi}_{BA}\rho_{B_0} \\ \dfrac{\partial \rho_A}{\partial t} + \dfrac{\partial}{\partial x}(\rho_A \boldsymbol{v}_A) = \dot{\chi}_{AB}\rho_{A_0} \end{cases}$$

Suppose that in both media the spherical part of Cauchy stress tensor

depends on $\xi_{AB} = 1 - \dfrac{\rho_{AB}}{\rho_{AB}^{(0)}}$

$$\begin{cases} \sigma_B = \sigma_B\left(1 - \dfrac{\rho_B}{\rho_B^{(0)}}\right) \\ \sigma_A = \sigma_A\left(1 - \dfrac{\rho_A}{\rho_A^{(0)}}\right) \end{cases}$$

Density ρ_B we shall seek in the form

$$\rho_B = \rho_{B1} + \tilde{\rho}_B$$

Here is

$$\frac{\partial \rho_{B1}}{\partial t} = -\dot{\chi}_{BA}\rho_{B_0} \Rightarrow \rho_{B1} = -\chi_{BA}\rho_{B_0} + C_B$$

From initial conditions at $t = 0$ we have

$$\rho_{B1}(0) = \rho_{B0}, \qquad \chi_{BA}(0) = 0, \qquad \mathrm{C_B} = \rho_{B0}$$

Then

$$\rho_{B1} = (1 - \chi_{BA})\rho_{B_0}$$

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For $\tilde{\rho}_B$ we have

$$\frac{\partial \tilde{\rho}_B}{\partial t} + \frac{\partial}{\partial x}\left[(\rho_{B1} + \tilde{\rho}_B)\boldsymbol{v}_B\right] = 0$$

At the first approximation by neglecting of $\tilde{\rho}_B$ in comparison with ρ_{B1}

$$\frac{\partial \tilde{\rho}_B}{\partial t} = -\frac{\partial}{\partial x}\left[\rho_{B1}\boldsymbol{v}_B\right] = -\frac{\partial}{\partial x}\left[(1-\chi_{BA})\boldsymbol{v}_B\right]\rho_{B0}$$

If χ_{BA} is not depending on x : $\dot{\tilde{\rho}}_B = -\rho_{B0}(1-\chi_{BA})\dot{\varepsilon}_B$

$$\tilde{\rho}_B = -\rho_{B0}\int_0^t (1-\chi_{BA})\dot{\varepsilon}_B d\tau = -\rho_{B0}\left[(1-\chi_{BA})\varepsilon_B(x,t) + \int_0^t \dot{\chi}_{BA}\varepsilon_B d\tau\right]$$

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For ρ_B

$$\rho_B = \rho_{B0}\left[1-\chi_{BA}-(1-\chi_{BA})\varepsilon_B(x,t)-\int_0^t \dot{\chi}_{BA}\varepsilon_B d\tau\right]$$

Then the equation of state for medium B

$$\sigma_B = \sigma_B\left[\chi_{BA}+(1-\chi_{BA})\varepsilon_B(x,t)+\int_0^t \dot{\chi}_{BA}\varepsilon_B d\tau\right]$$

Without of sources $\chi_{BA}=0$ the equation of state is $\sigma_B(\varepsilon_B)$

Let's note that the density ρ_B can turn into 0 at moment t^*

$$(1-\chi_{BA})(1-\varepsilon_B(x,t))-\int_0^{t^*} \dot{\chi}_{BA}\varepsilon_B d\tau = 0$$

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For ρ_{A1}

$$\rho_{A1} = \chi_{AB}\rho_{A0} + C_A \qquad \rho_{A1}\big|_{t=0} = 0 \quad \Rightarrow C_A = 0$$

$$\rho_{A1} = \chi_{AB}\rho_{A0}$$

$$\frac{\partial\tilde{\rho}_A}{\partial t} = -\frac{\partial}{\partial x}\left[\rho_{A1}\boldsymbol{v}_A\right] = -\frac{\partial}{\partial x}\left[\chi_{AB}\boldsymbol{v}_A\right]\rho_{A0} = -\rho_{A0}\dot{\varepsilon}_A\chi_{AB}$$

$$\rho_A = \rho_{A1} + \tilde{\rho}_A = \rho_{A0}\left[\chi_{AB}(1-\varepsilon_A) + \int_0^t \dot{\chi}_{AB}\varepsilon_A d\tau\right]$$

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Thus there are two equations for the determination of the densities of components A and B for given source χ_{BA}

$$\frac{\rho_B}{\rho_{B0}} = (1-\chi_{BA})(1-\varepsilon_B(x,t)) - \int_0^t \dot{\chi}_{BA}\varepsilon_B d\tau$$

$$\frac{\rho_A}{\rho_{B0}} = \chi_{BA}(1-\varepsilon_A(x,t)) + \int_0^t \dot{\chi}_{BA}\varepsilon_A d\tau$$

Equations of state for two components

$$\sigma_B = \sigma_B\left(\varepsilon_B(x,t) + \chi_{BA}(1-\varepsilon_B(x,t)) + \int_0^t \dot{\chi}_{BA}\varepsilon_B d\tau \right)$$

$$\sigma_a = \sigma_A\left(\chi_{AB}\varepsilon_A(x,t) + 1 - \chi_{BA} - \int_0^t \dot{\chi}_{AB}\varepsilon_A d\tau \right)$$

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or in case of linearization

$$\sigma_B \approx k_B \left[\varepsilon_B(x,t) + \chi_{BA}(1 - \varepsilon_B(x,t)) + \int_0^t \dot{\chi}_{BA} \varepsilon_B \, d\tau \right]$$

$$\sigma_a \approx k_A \left[\chi_{AB} \varepsilon_A(x,t) + 1 - \chi_{AB} - \int_0^t \dot{\chi}_{AB} \varepsilon_A \, d\tau \right]$$

Two main source terms

1.

$$\chi = \alpha \cdot (T - T_0)$$

$$\sigma = k_a(\varepsilon - \alpha \cdot (T - T_0))$$

$$\frac{\partial \tilde{\rho}_A}{\partial t} + \rho_{A_0} \cdot \dot{\varepsilon} = (\alpha T)^{\square} \rho_{A_0}$$

$$\tilde{\rho}_A = -\rho_{A_0}(\varepsilon - \alpha(T - T_0))$$

$$\sigma_A = k_A\left(1 - \frac{\rho_A}{\rho_{A_0}}\right) = k_A(\varepsilon - \alpha(T - T_0))$$

2.

$$\chi = \varepsilon \frac{n^{+}}{\kappa + n^{+}}$$

$$\tilde{\rho}_{A} = -\rho_{A_0} \left(\varepsilon - \varepsilon \frac{n^{+}}{\kappa + n^{+}}\right) = -\rho_{A_0} \varepsilon \frac{\kappa}{\kappa + n^{+}}$$

$$\sigma_{A} = E_{A} \varepsilon \frac{\kappa}{\kappa + n^{+}}$$

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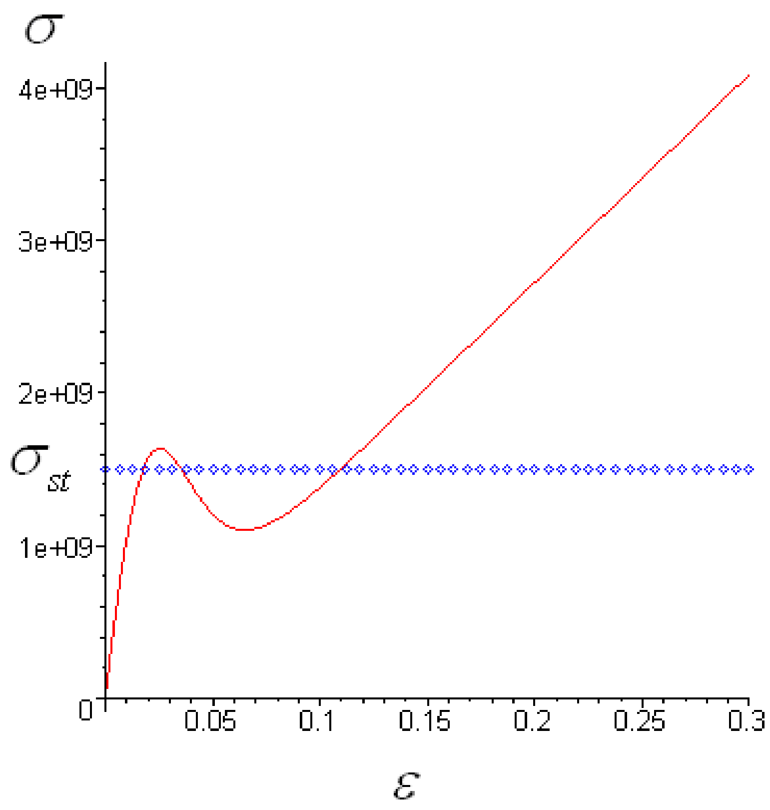
Static diagram



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Two-component one-dimensional model of a thermo-elastic material of complex structure

Basic equations of one-dimensional model

$$\begin{cases} \dfrac{\partial \sigma_1}{\partial x} - \rho_1 \dfrac{\partial^2 u_1}{\partial t^2} + \rho_1 F_1^e - R = 0 \\ \dfrac{\partial \sigma_2}{\partial x} - \rho_2 \dfrac{\partial^2 u_2}{\partial t^2} + \rho_2 F_2^e + R = 0 \end{cases}$$

$$\begin{cases} \lambda_1 \dfrac{\partial^2 T_1}{\partial x^2} - \rho_1 c_1 \dfrac{\partial T_1}{\partial t} = E_1 \alpha_1 \theta_0 \dfrac{\partial^2 u_1}{\partial x \partial t} - \rho_1 b_1 - \kappa \left(T_1 - T_2 \right) \\ \lambda_2 \dfrac{\partial^2 T_2}{\partial x^2} - \rho_2 c_2 \dfrac{\partial T_2}{\partial t} = E_2 \alpha_2 \theta_0 \dfrac{\partial^2 u_2}{\partial x \partial t} - \rho_2 b_2 + \kappa \left(T_1 - T_2 \right) \end{cases}$$

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$$\sigma_k = E_k\left(\varepsilon_k - \alpha_k(\theta_k - \theta_0)\right) \qquad \text{k=1,2}$$

$$\begin{cases} E_1 \dfrac{\partial^2 u_1}{\partial x^2} - \rho_1 \dfrac{\partial^2 u_1}{\partial t^2} - E_1\alpha_1 \dfrac{\partial T_1}{\partial x} + \rho_1 F_1^e - R = 0 \\ E_2 \dfrac{\partial^2 u_2}{\partial x^2} - \rho_2 \dfrac{\partial^2 u_2}{\partial t^2} - E_2\alpha_2 \dfrac{\partial T_2}{\partial x} + \rho_2 F_2^e + R = 0 \end{cases}$$

Here $$R = R_1(u_1 - u_2) + R_2\left(\frac{\partial u_1}{\partial t} - \frac{\partial u_2}{\partial t}\right)$$

$T_k = \theta_k - \theta_0,$ θ_0 - the reference temperature

$\boldsymbol{R}$ - the force of intercomponental mechanical interaction

$Q = \kappa(T_1 - T_2)$ – intercomponental temperature interactions.

On schedules T_2, V_2 are distinctly observed oscillations which frequency is equal to frequency of partial fluctuations of the second component on elastic bonds R_1 and it is approximately estimated as $f = \frac{1}{2\pi}\sqrt{\frac{R_1}{\rho_2}} \approx \frac{1}{2\pi} 0.741 \cdot 10^9 \approx 1.18 \cdot 10^8 \ Hz$

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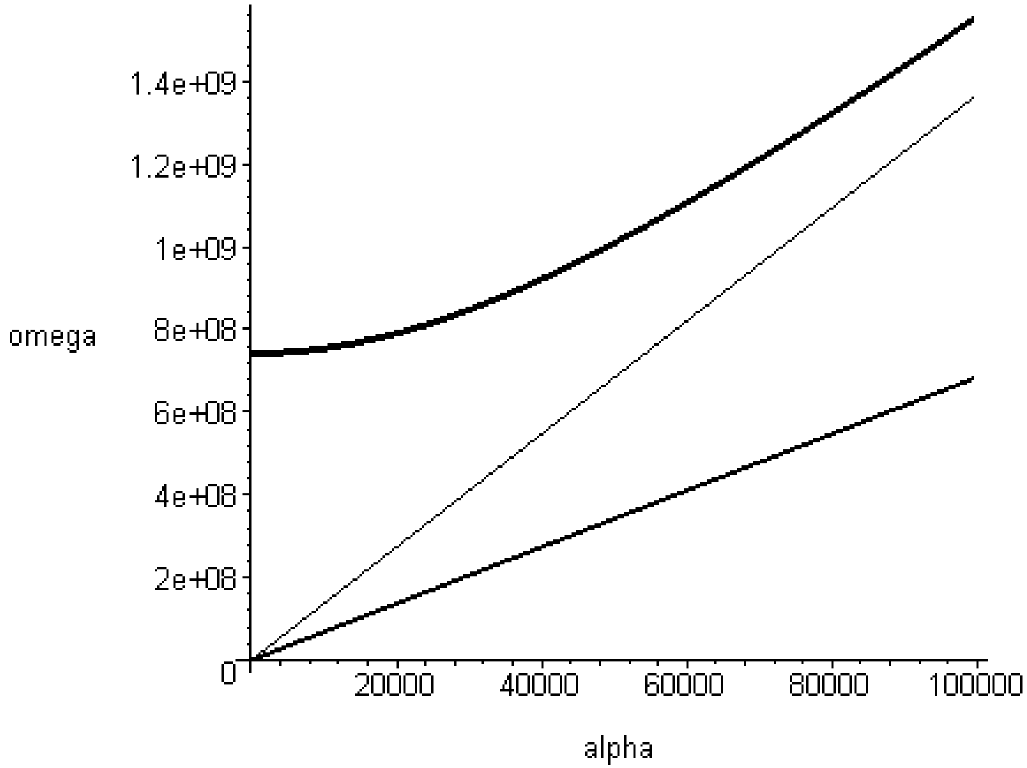

Fig. 6 Dispersive curves

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Features of behaviour of two-component model under the action of non-stationary loadings

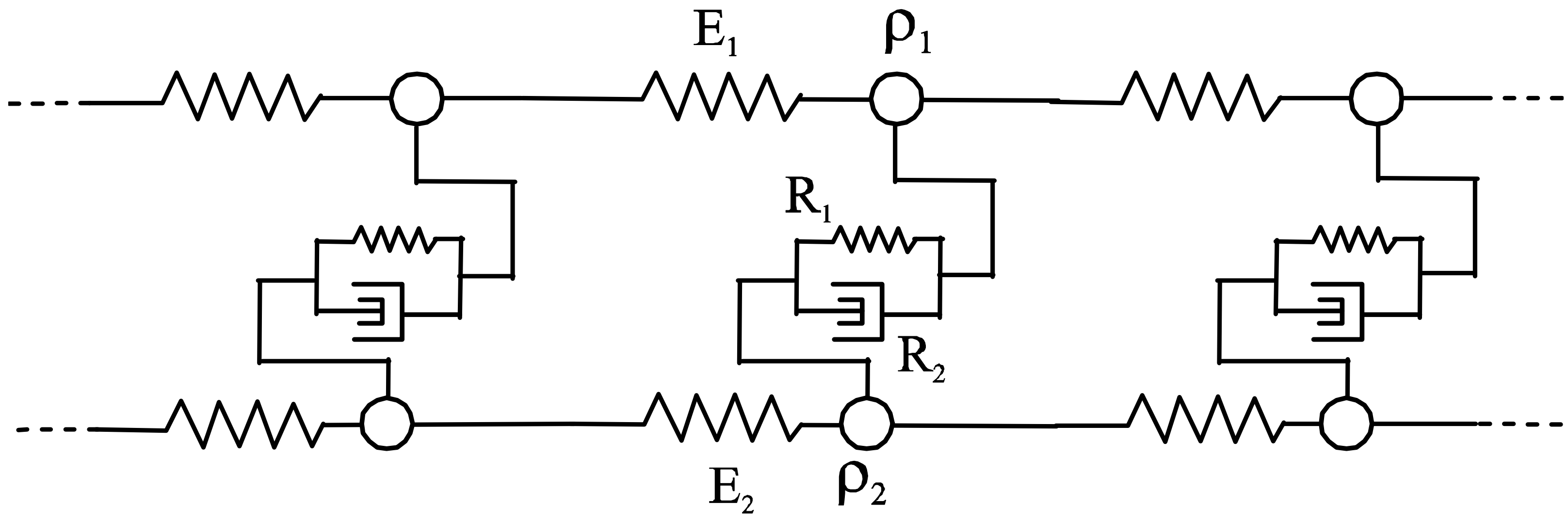

Fig. 3 The general view of the structural - rheological model

$$R = R_1(u_1 - u_2) + R_2\left(\frac{\partial u_1}{\partial t} - \frac{\partial u_2}{\partial t}\right)$$

- force interaction of the material components is supposed to be linear elastic-viscous.

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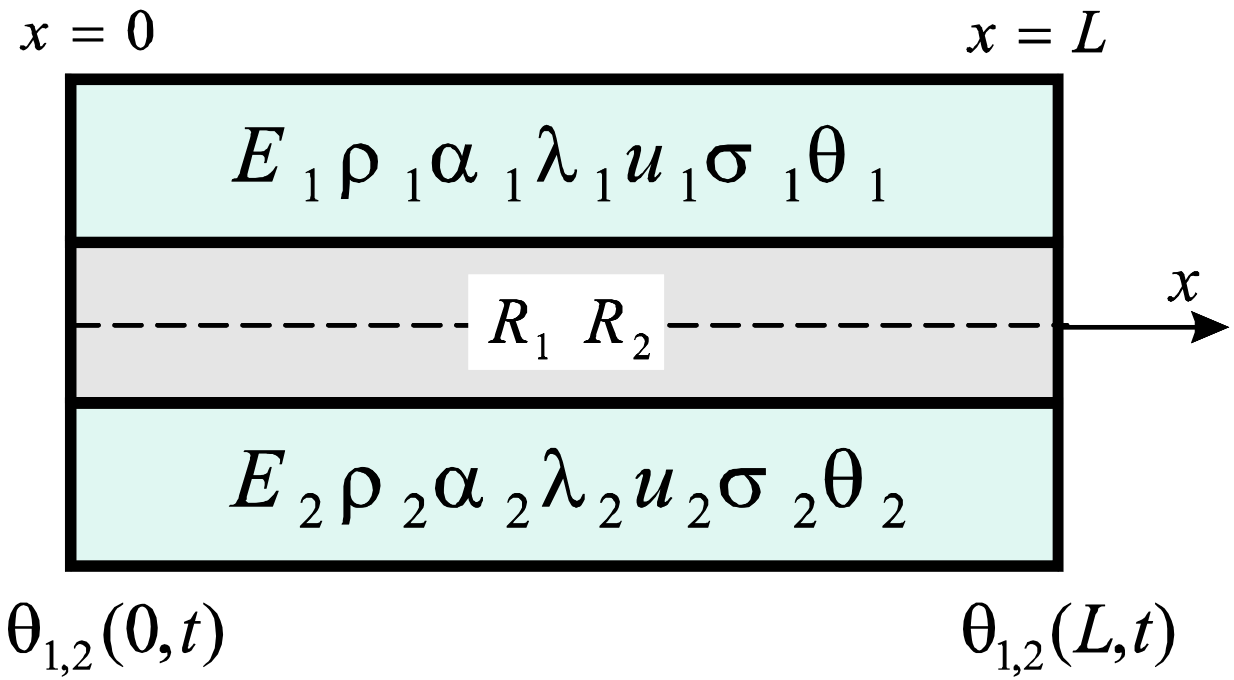
Model of deformable two-componental thermoelastic one-dimensional rod



Problem of statement of boundary conditions?

Pulse temperature loading of a semi-conductor crystal (a Si sample)

At very low temperatures ($\approx 4,2^0 K$) it is possible the generation of wave fluctuations of temperature

$$\lambda_1 \approx 0, \qquad \lambda_2 \neq 0$$

Density of silicon is

$$\rho \approx 2.33 \cdot 10^3 \ \frac{kg}{m^3}$$

The atom of silicon has 14 electrons.

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The distribution of a wave thermo-mechanical pulse in a crystal semi-conductor Si sample.

Estimation of parameters of the two-component model

- Specific thermal capacity (at 20 – 100 0C) $\approx 800\ J/kg\ K$

$$c_1 \approx 0.001 \frac{J}{kg\ K},\ \ c_2 \approx 1 \frac{J}{kg\ K}$$

- Heat conductivity (at 25^{0}C) $\approx 84 \div 126 \frac{W}{m\ K}$

$$\lambda_1 \approx 0 \frac{W}{m\ K},\ \ \lambda_2 \neq 0 \frac{W}{m\ K}$$

- Temperature factor of linear expansion for silicon at normal temperature

$$\alpha \approx 2.33 \cdot 10^{-6}\ 1/K \qquad E \approx 109 \cdot 10^9 \frac{N}{m^2}$$

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Temperature factor of linear expansion for silicon at normal temperature $\alpha \approx 2.33 \cdot 10^{-6}\ 1/K$.

At temperature lower $120\ K$ this factor becomes negative.

For modelling (behind absence of more detailed information) it is accepted $\alpha_1 \approx 2.33 \cdot 10^{-9}\ 1/K,\ \alpha_2 \approx 2.33 \cdot 10^{-6}\ 1/K$

The elasticity module of silicon at normal temperature is $E \approx 109 \cdot 10^{9}\ \dfrac{N}{m^2}$.

For two-component model it is accepted .

$E_2 \neq 0$ means presence of own elasticity arising due to polarization of electric fields at displacement of electronic environments of atoms of silicon.

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Density of silicon is $\rho \approx 2.33 \cdot 10^3 \dfrac{kg}{m^3}$. The atom of silicon has 14 electrons. They settle down on 3 environments: 2-8-4, on last environment are 4 electrons. Most likely, these last electrons "form" the second deformable component of considered two-component model. Probably, this statement will demand correction. Generally speaking, it is possible to think of formation of the second component by amount of electrons from 1 up to 14. Weight of a nucleus of atom of silicon is . Weight of electrons involved in formation of the second components is $9.11 \cdot 10^{-31} \div 1.275 \cdot 10^{-29}\ kg$.

According to it the densities of components of model could be accepted as

$\rho_1 \approx 2.33 \cdot 10^3 \dfrac{kg}{m^3}, \quad \rho_2 \approx 0.0454 \div 0.635 \dfrac{kg}{m^3}$. At the rate of 4 electrons, forming the second component we have the density of the component $\rho_2 \approx 0.182 \dfrac{kg}{m^3}$. Parameters of model accepted here to consideration have ratio among themselves, close to a reality.

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Modules of elasticity and mass density define the next velocities of distribution of wave pulses on both components $v_2 \approx 13700 > v_1 \approx 6850 \quad v_2 = 2v_1$.

Concerning factor of an intercomponental temperature exchange there are no data (reasons), most likely in conditions of the absence of free electrons for modelling it is necessary to accept factor κ close to zero $\kappa \approx 0 \frac{W}{m^3\ K}$.

At a choice of factor of intercomponental force interaction R_1, probably, it is necessary to be guided $\rho_2 \approx 0.182 \frac{kg}{m^3}$ by value of the frequency $f \approx 1.18 \cdot 10^8\ Hz$ of partial fluctuations of the second component respecting the first. If to accept and be guided by frequency for factor of elastic intercomponental interaction the estimation $R_1 \approx 1 \cdot 10^{17} \frac{N}{m^4}$ turns out. Concerning factor of force viscous interaction of the components R_2 while is not present precise reasons (assumptions), therefore we shall put $R_2 \approx 10^3 \frac{N\ s}{m^4}$.

It is supposed, that the loading of an one-dimensional sample

$0 < x < l \approx 5.5 \cdot 10^{-3}\ m$ at an end face $x = 0$ it is carried out by a short-term temperature pulse

$$T_1\Big|_{x=0} = T_{10}\big(H(t) - H(t-\tau)\big), \quad \left.\frac{\partial T_2}{\partial x}\right|_{x=0} = 0$$

$$T_2\Big|_{x=0} = T_{20}\big(H(t) - H(t-\tau)\big), \quad \left.\frac{\partial T_1}{\partial x}\right|_{x=0} = 0$$

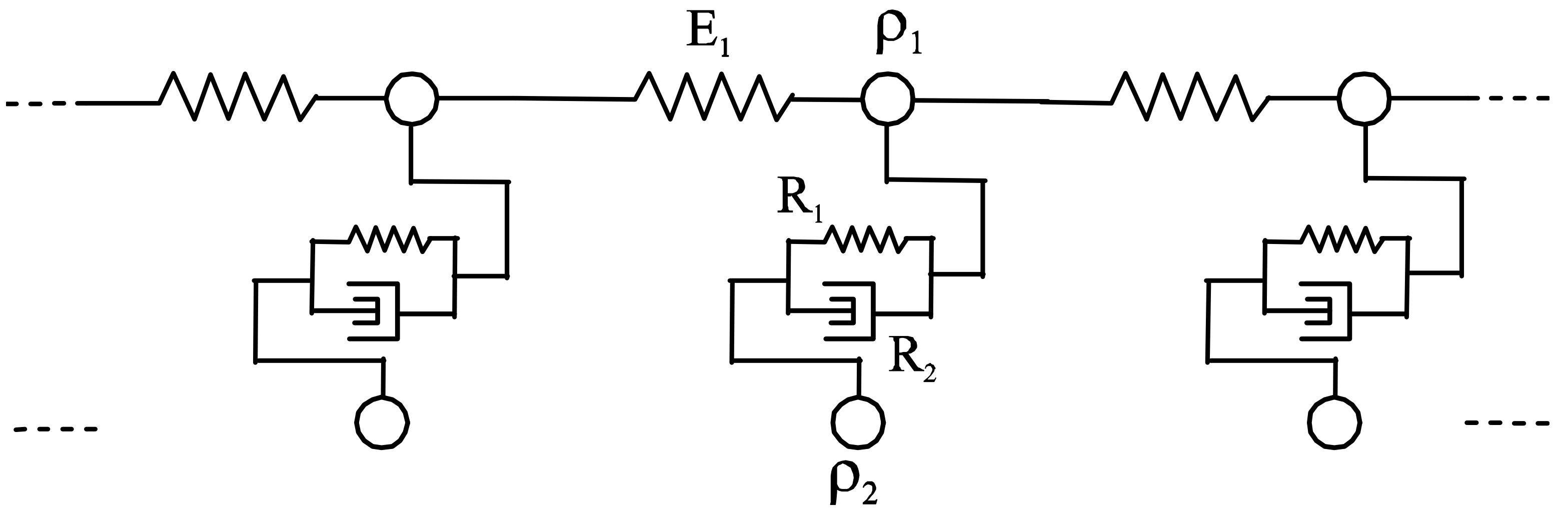

Fig.4 Structural - rheological circuit of one-dimensional two-component object with properties of non-metal

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For modelling it is supposed to accept $\alpha_1 = 2.33 \cdot 10^{-9}, \alpha_2 = 2.33 \cdot 10^{-6}$

The temperature pulse $T_1\big|_{x=0} = T_{10}\left(H(t) - H(t-\tau)\right), \quad \left.\frac{\partial T_2}{\partial x}\right|_{x=0} = 0$

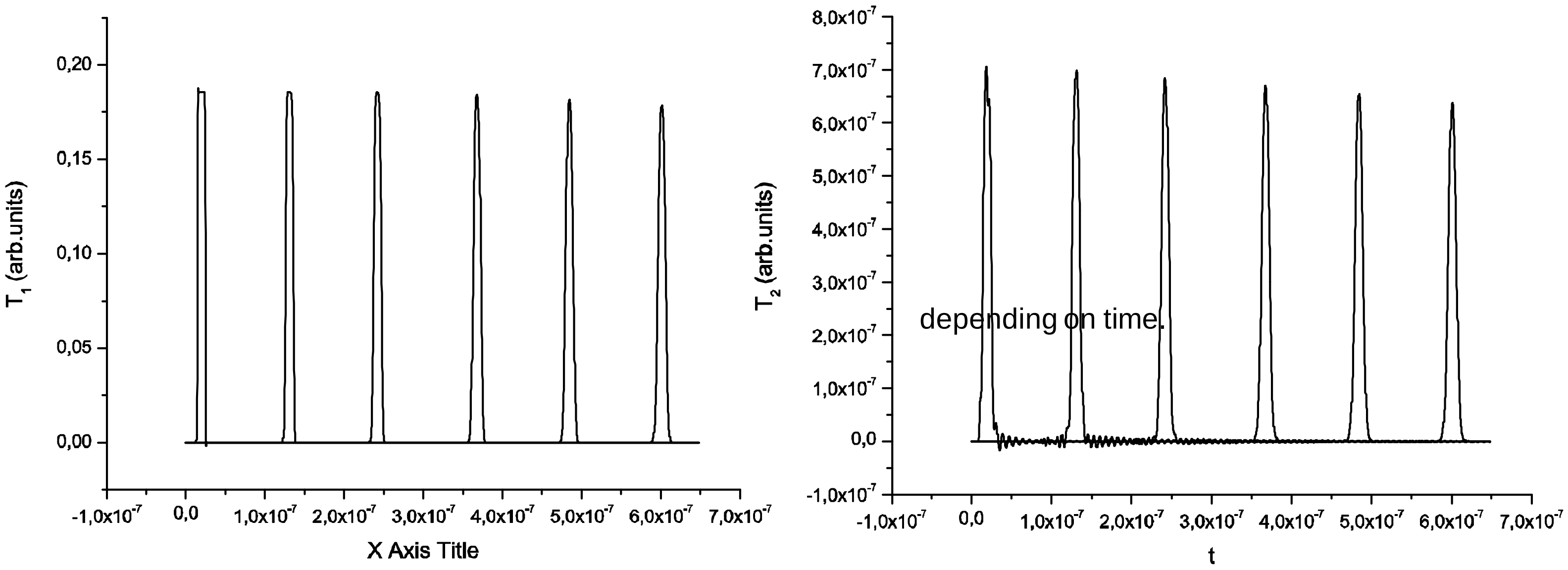

Fig. 4 Temperatures of the first and second components of sequence of sections $x \approx 0,\ x \approx 1,\ x \approx 2,\ x \approx 3,\ x \approx 4,\ x \approx 5\,mm$

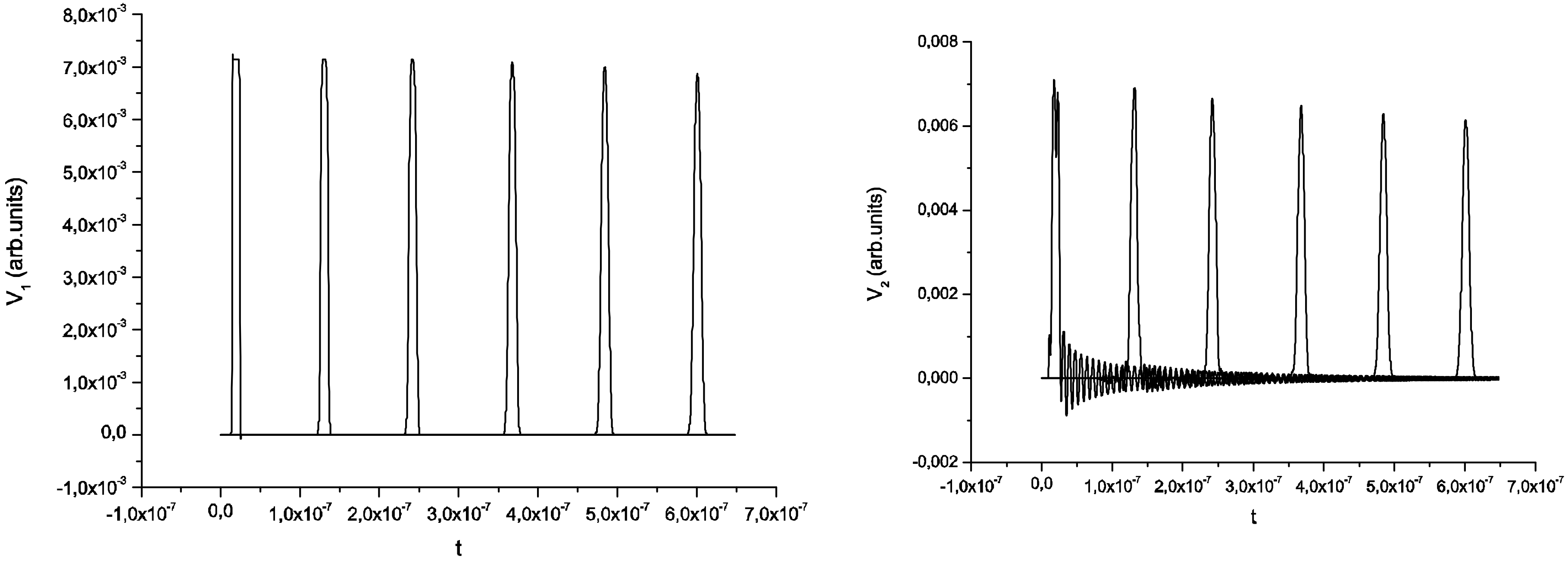

Fig. 5 Velocities of the first and second components of sequence of sections $x \approx 0,\ x \approx 1,\ x \approx 2,\ x \approx 3,\ x \approx 4,\ x \approx 5\,mm$ depending on time.

On schedules T_2, V_2 are distinctly observed oscillations which frequency is equal to frequency of partial fluctuations of the second component on elastic bonds R_1 and it is approximately estimated as $f = \frac{1}{2\pi}\sqrt{\frac{R_1}{\rho_2}} \approx \frac{1}{2\pi} 0.741 \cdot 10^9 \approx 1.18 \cdot 10^8\ Hz$

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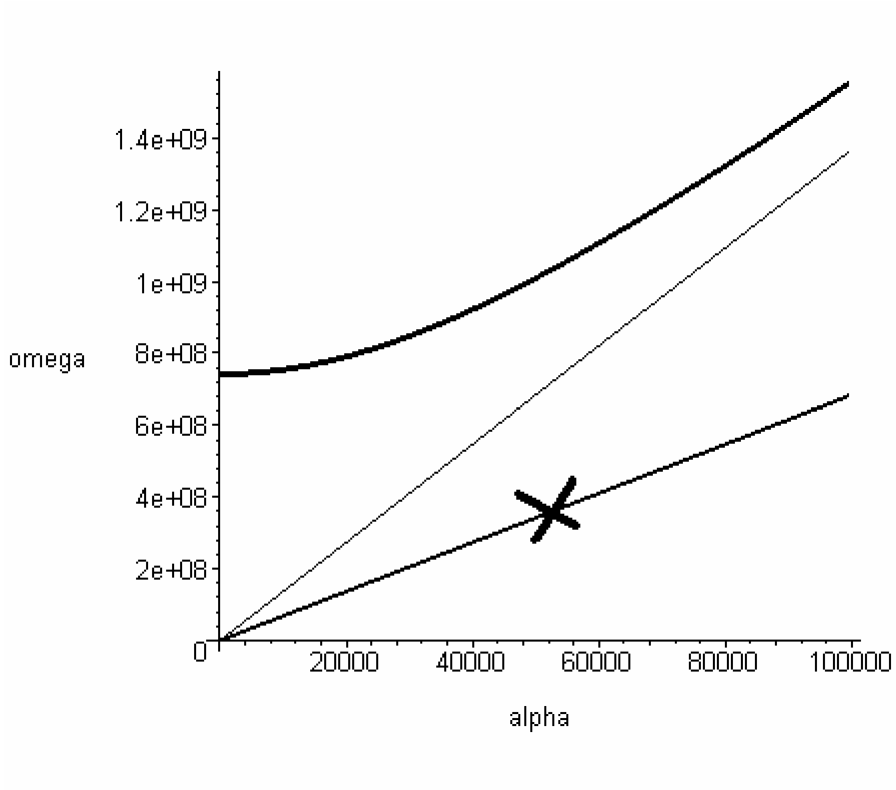

Fig. 6 Dispersive curves

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The speed of distribution of a wave temperature pulse in the first component according to schedule Fig. of 4 is more as the value $v_1 = \sqrt{\frac{E_1}{\rho_1}}$. The reason consists that at adiabatic process of a heat transfer $\lambda_1 \approx 0$, speed is approximately determined by expression that for the accepted values of parameters of model makes $\tilde{v}_1 \approx \sqrt{\frac{E_1}{\rho_1}}\sqrt{1+\alpha_1\frac{E_1\alpha_1\theta_0}{\rho_1 c_1}}$ $\tilde{v}_1 \approx 1.23 v_1$

In what kind the energy of laser excitation is transferred through a sample to measuring bolometer in section $x = l \approx 5\,mm$

kinetic energy of the first component $K_1 = \frac{1}{2}\rho_1 V_1^{\ 2} \approx 0.57 \cdot 10^{-1}$

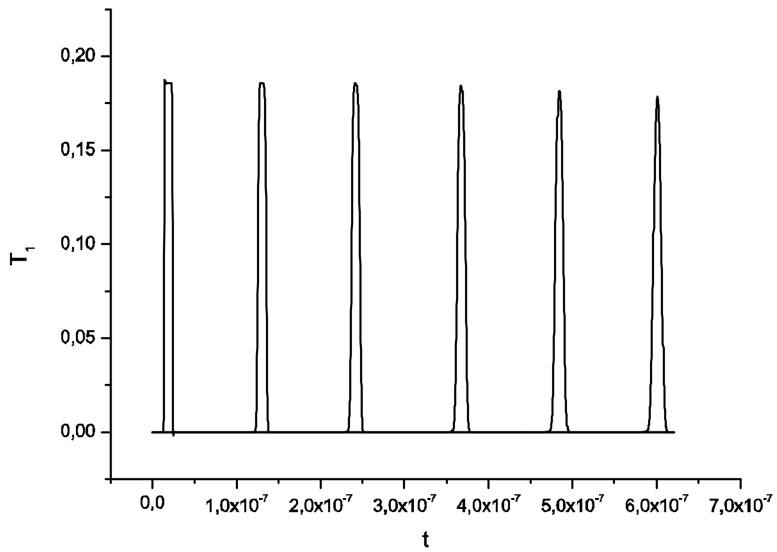
- kinetic energy of the second component $K_2 = \frac{1}{2}\rho_2 V_2^{\ 2} \approx 0.33 \cdot 10^{-5}$

- thermal energy of the first component $E_{T1} = c_1 \rho_1 T_1 \approx 0.42$

- thermal energy of the second component $E_{T2} = c_2 \rho_2 T_2 \approx 0.13 \cdot 10^{-6}$

For the reduced size of force interaction of components $R_1 \approx 1 \cdot 10^{16}$

instead of $R_1 \approx 1 \cdot 10^{17}$



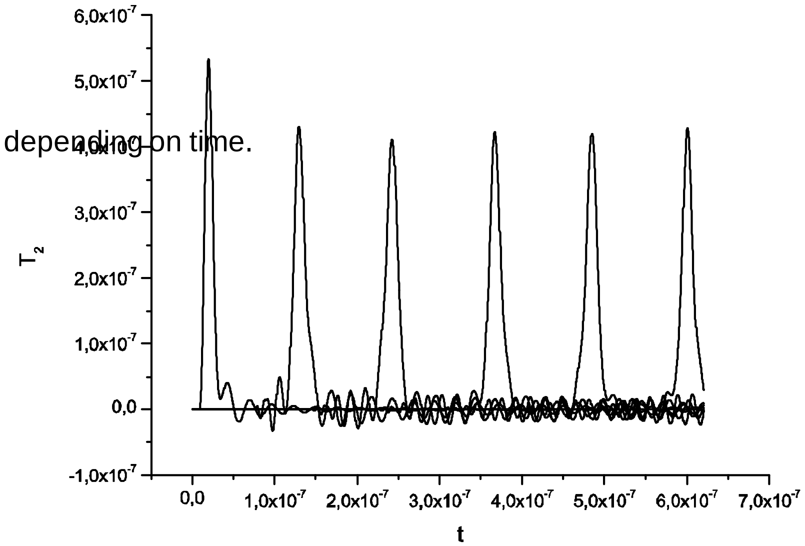


Fig. 7 Temperatures of the first and second components
of sequence of sections $x \approx 0,\ x \approx 1,\ x \approx 2,\ x \approx 3,\ x \approx 4,\ x \approx 5\,mm$
depending on time.

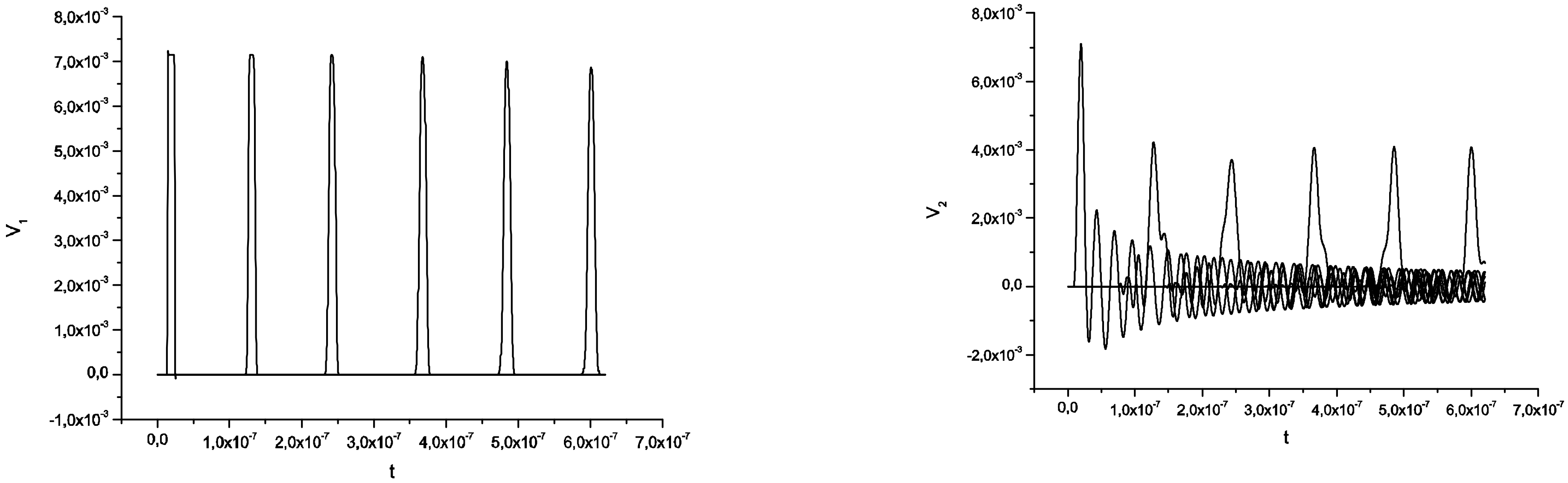


Fig. 8 Velocities of the first and second components of sequence of sections depending on time.

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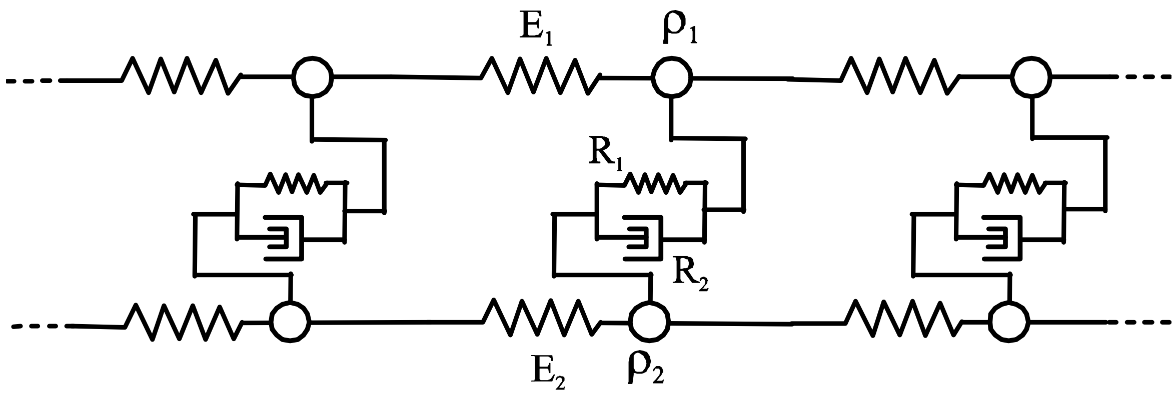
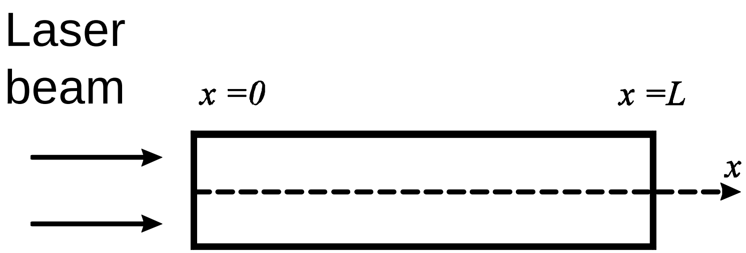
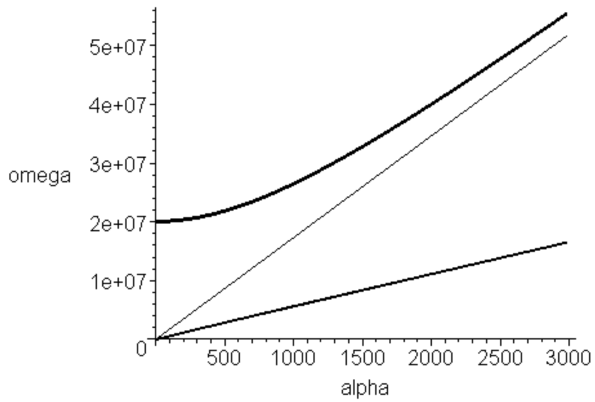
Use of the two-componental model of thermoelastic bodies for the analysis of propagation of wave pulses in a Si-sample, excited by short laser impact







Dispersion curves

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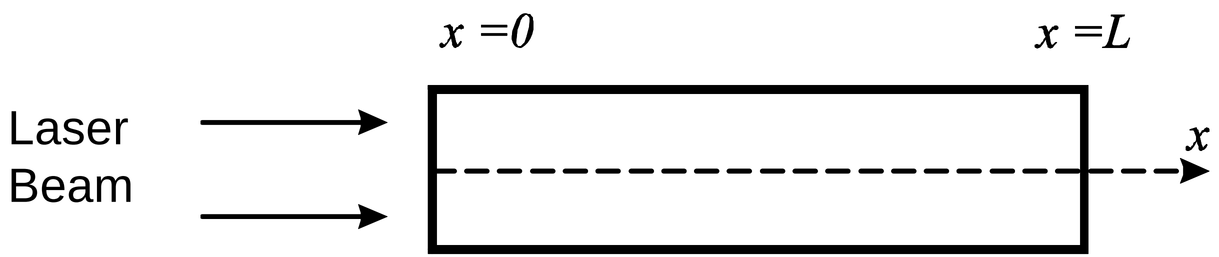
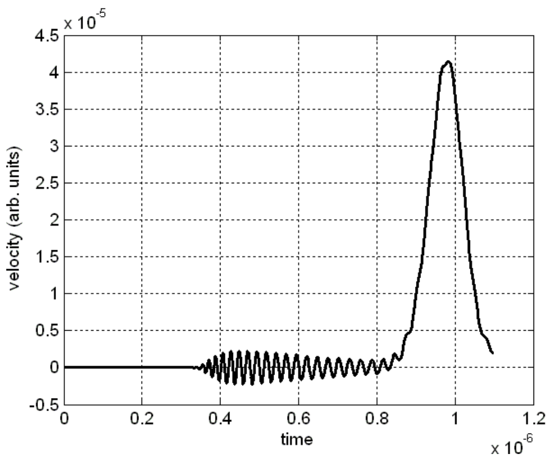
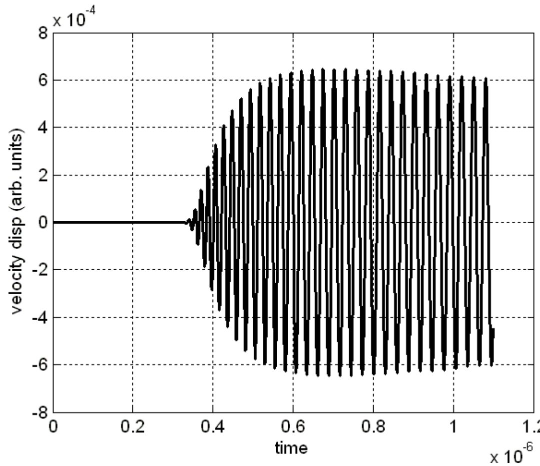
Use of the two-componental model of thermoelastic bodies for the analysis of propagation of wave pulses in a Si-sample, excited by short laser impact



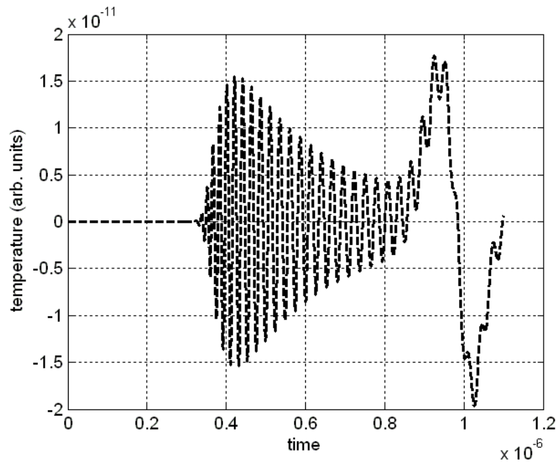
Temperature loading



Speed of end sections of a sample



Difference of speeds of components In end section



Temperature of end sections of a sample

Systems of thermoelasticity- heat and mass transfer-intercomponent exchange equations

$$
\begin{cases}
\dfrac{\partial \sigma_1}{\partial x} = \rho_0 \dfrac{\partial^2 u_1}{\partial t^2}, \qquad \sigma_1 = E_0 \varepsilon_1^{(m)} \dfrac{1}{1 + n^+ / \kappa}, \\
\varepsilon_1^{(m)} = \varepsilon_1 - \alpha_1 (T_1 - T_0), \qquad \varepsilon_1 = \dfrac{\partial u_1}{\partial x}, \\
\dfrac{\partial n^-}{\partial t} = \dfrac{\partial}{\partial x}\left(D_n (T_2)(1 - b\varepsilon_1^{(m)}) \dfrac{\partial n^-}{\partial x} \right) + \dfrac{\partial}{\partial x}\left(D_T (n^-)(1 - b\varepsilon_1^{(m)}) \dfrac{\partial T_2}{\partial x} \right) - \\
\qquad - \alpha(\varepsilon_1, T_1) n^- + \beta(\varepsilon_1, T_1) n^+, \\
\dfrac{\partial n^+}{\partial t} = \boxed{\alpha(\varepsilon_1, T_1) n^- - \beta(\varepsilon_1, T_1) n^+} - \dfrac{\partial}{\partial x}\left(n^+ \dfrac{\partial u_1}{\partial t} \right), \\
c_1 \rho_0 \dfrac{\partial T_1}{\partial t} = \lambda_1 \dfrac{\partial^2 T_1}{\partial x^2} - E_0 \dfrac{1}{1 + n^+ / \kappa} \alpha_1 T_0 \dfrac{\partial \varepsilon_1}{\partial t} + \lambda_0 (T_1 - T_2), \\
c_2 m_2 n^- \dfrac{\partial T_2}{\partial t} = \lambda_2 \dfrac{\partial^2 T_2}{\partial x^2} + \lambda_n \dfrac{\partial}{\partial x}\left(D_n (T_2)(1 - b\varepsilon_1^{(m)}) \dfrac{\partial n^-}{\partial x} \right) - \lambda_0 (T_1 - T_2).
\end{cases}
$$

Numerical simulation

1

$$\sigma_1(0,t)=(1-e^{-\lambda t})\,\sigma_0\,, \qquad \alpha = Const, \qquad \beta = Const$$

$$u_1(x,0)=0, \quad u_1(1,t)=0, \quad T = const\,,$$

$$n^-(x,0)=C\begin{cases}1, & x\in\bigcup(a_i,b_i),\\ 0, & x\notin\bigcup(a_i,b_i),\end{cases} \qquad n^+(x,0)=0,$$

$$\frac{\partial n^-}{\partial x}(0,t)=0, \qquad \frac{\partial n^+}{\partial x}(0,t)=0,$$

$$\frac{\partial n^-}{\partial x}(1,t)=0, \qquad \frac{\partial n^+}{\partial x}(1,t)=0.$$

$$x_1=0.035,\; x_3=0.044,\; x_4=0.06,\; x_5=0.07,\; x_2=0.14$$

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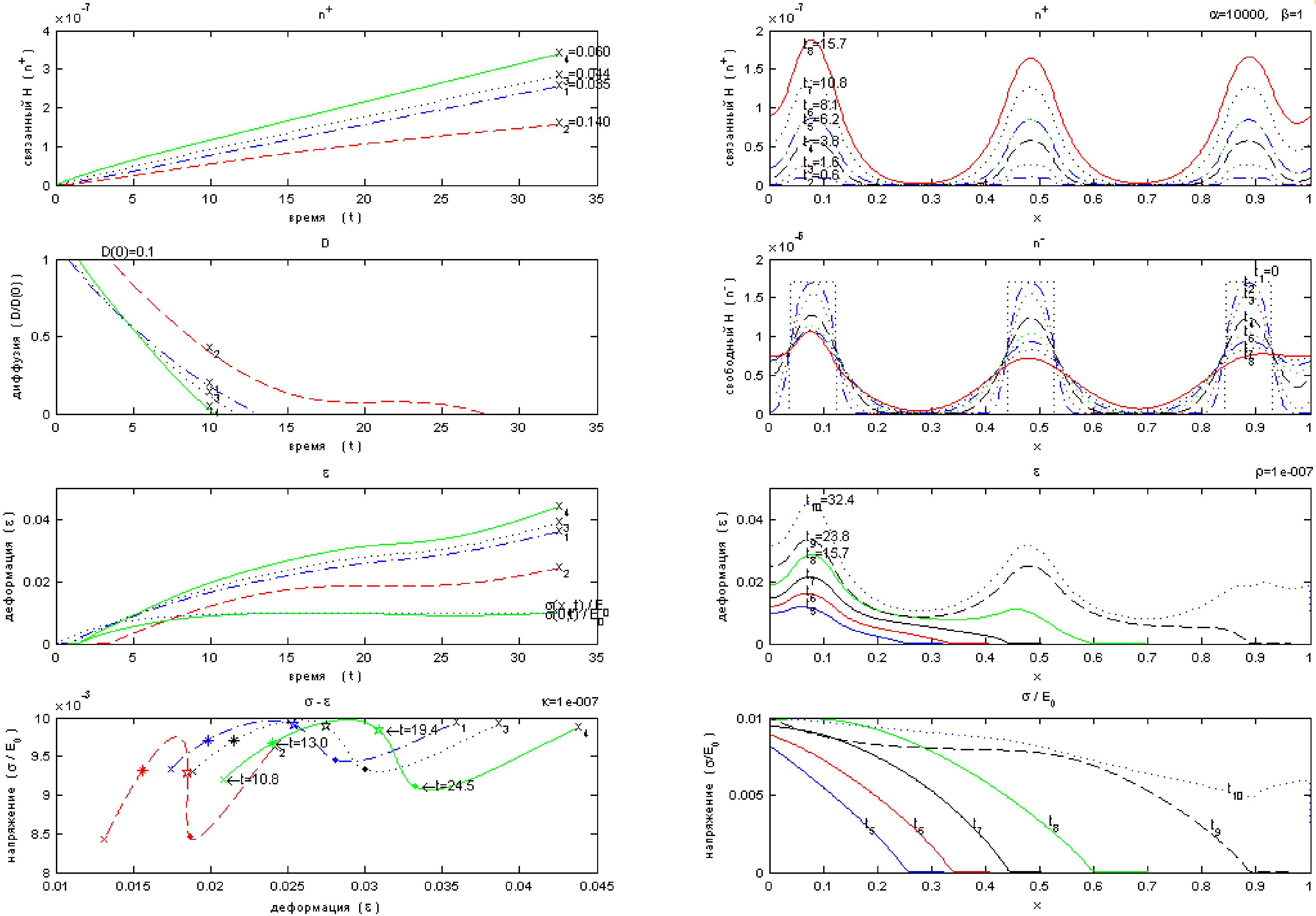
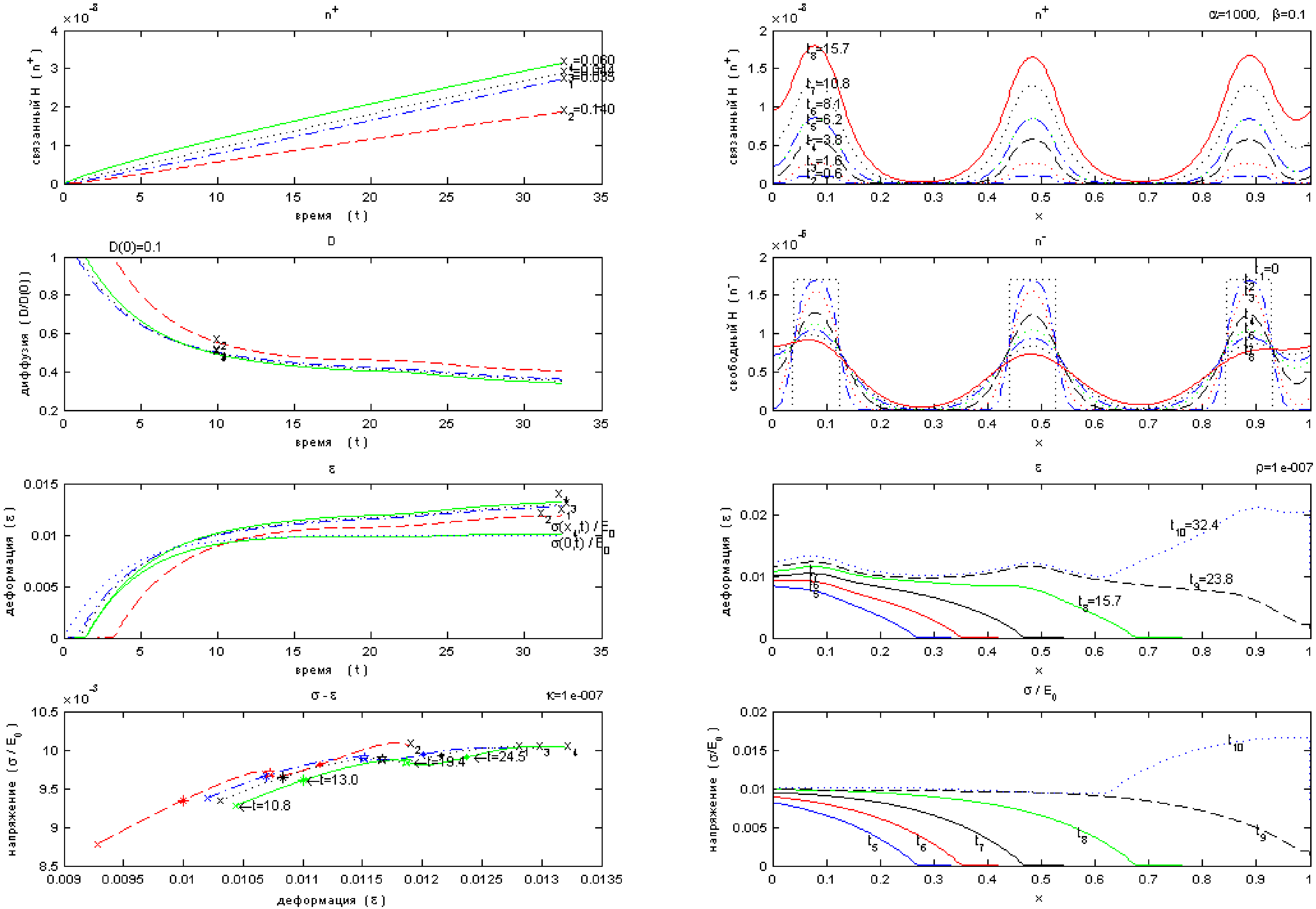
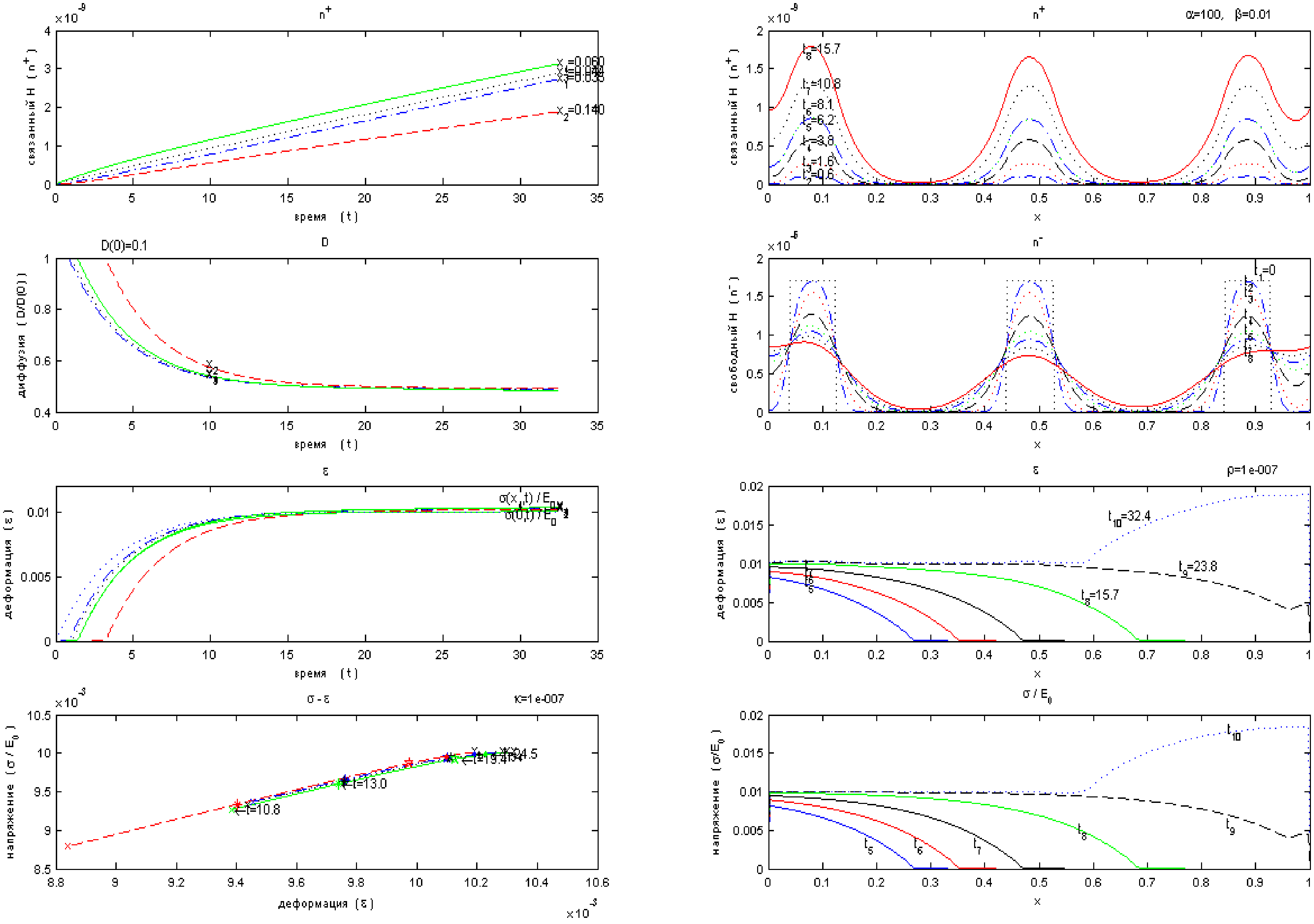
α=10000, β=1


n+
связанный H (n+)
время (t)
x4=0.060
x3=0.044
x1=0.035
x2=0.140
α=10000, β=1
x
D
D(0)=0.1
диффузия (D/D(0))
n-
свободный H (n-)
t1=0
ε
деформация (ε)
ρ=1e-007
t10=32.4
t9=23.8
t8=15.7
σ(x,t)/E0
σ(0,t)/E0
σ - ε
κ=1e-007
напряжение (σ / E0)
деформация (ε)
t=19.4
t=13.0
t=10.8
t=24.5
σ / E0
напряжение (σ/E0)

α=1000, β=0.1


n+
связанный H (n+)
время (t)
x1=0.060
x4=0.944
x3=0.835
x2=0.140
α=1000, β=0.1
t8=15.7
t7=10.8
t6=8.1
t5=6.2
t4=3.8
t3=1.6
t2=0.6
x
D
D(0)=0.1
диффузия (D/D(0))
n-
свободный H (n-)
t1=0
ε
ρ=1e-007
деформация (ε)
σ(x4,t) / E0
σ(0,t) / E0
t10=32.4
t9=23.8
t8=15.7
σ - ε
κ=1e-007
напряжение (σ / E0)
деформация (ε)
←t=10.8
←t=13.0
←t=19.4
←t=24.5
σ / E0
напряжение (σ/E0)
t5
t6
t7
t8
t9
t10

α=100, β=0.01


n+
связанный Н (n+)
время (t)
α=100, β=0.01
x
D(0)=0.1
D
диффузия (D/D(0))
n-
свободный Н (n-)
ε
деформация (ε)
ρ=1e-007
σ(x,t) / E0
σ(0,t) / E0
t10=32.4
t9=23.8
t8=15.7
σ - ε
κ=1e-007
напряжение (σ / E0)
деформация (ε)
t=10.8
t=13.0
σ / E0
напряжение (σ/E0)

$\boxed{2}$

$$\sigma_1(0, t) = \delta(t)\,,$$

$$u_1(x, 0) = 0, \quad u_1(1, t) = 0, \quad T = const \ ,$$

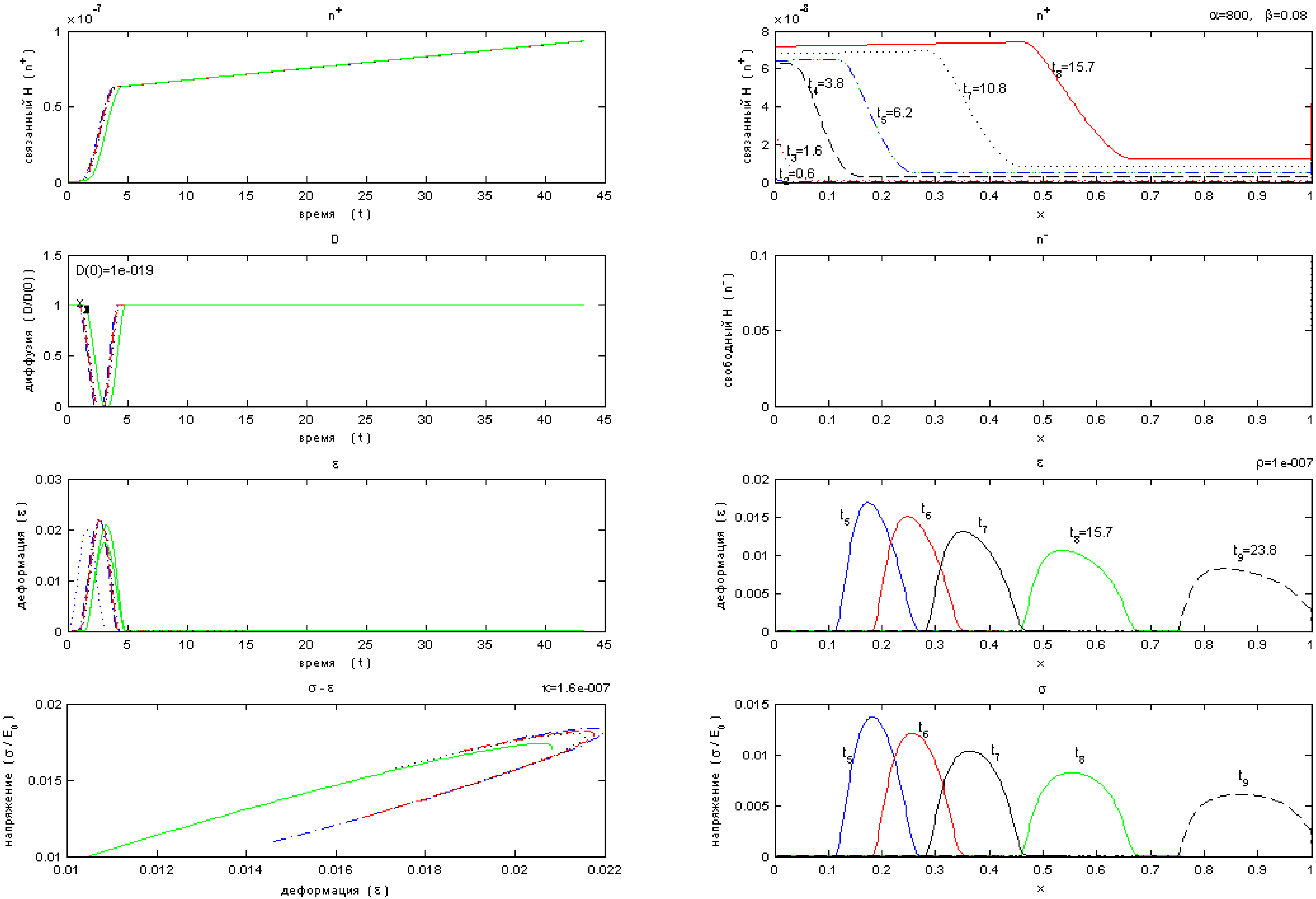
$$n^-(x, 0) = C\,, \qquad n^+(x, 0) = 0,$$

$$\frac{\partial n^-}{\partial x}(0, t) = 0, \qquad \frac{\partial n^+}{\partial x}(0, t) = 0,$$

$$\frac{\partial n^-}{\partial x}(1, t) = 0, \qquad \frac{\partial n^+}{\partial x}(1, t) = 0.$$

$$x_1 = 0.035, \ x_2 = 0.04, \ x_3 = 0.044, \ x_4 = 0.06, \ x_5 = 0.07$$

$\alpha=0.08+160\cdot\varepsilon,\ \beta=0.08$



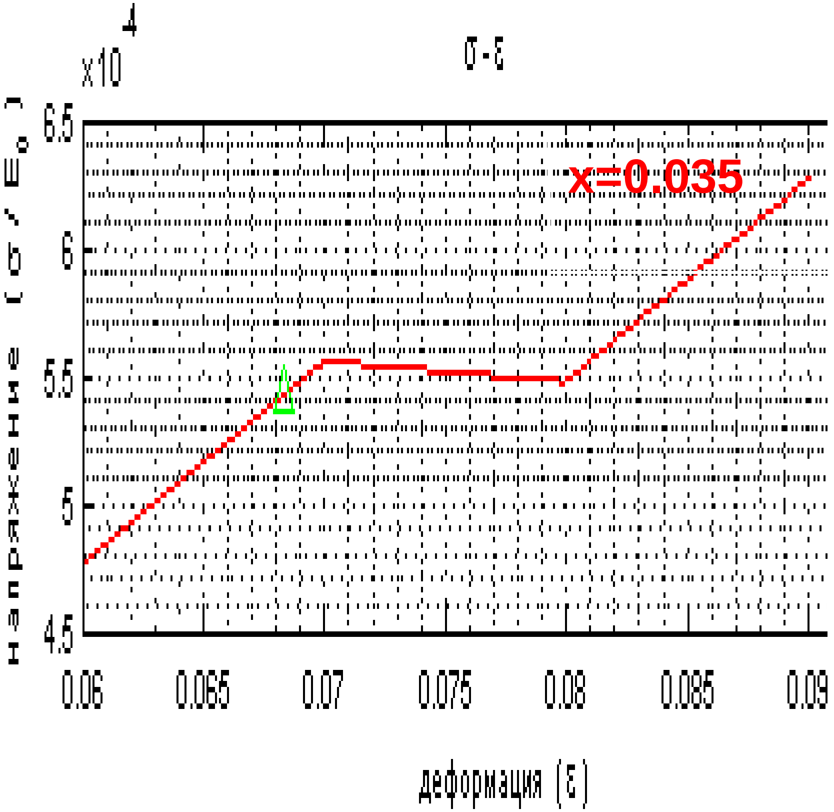
Improvement of material structure

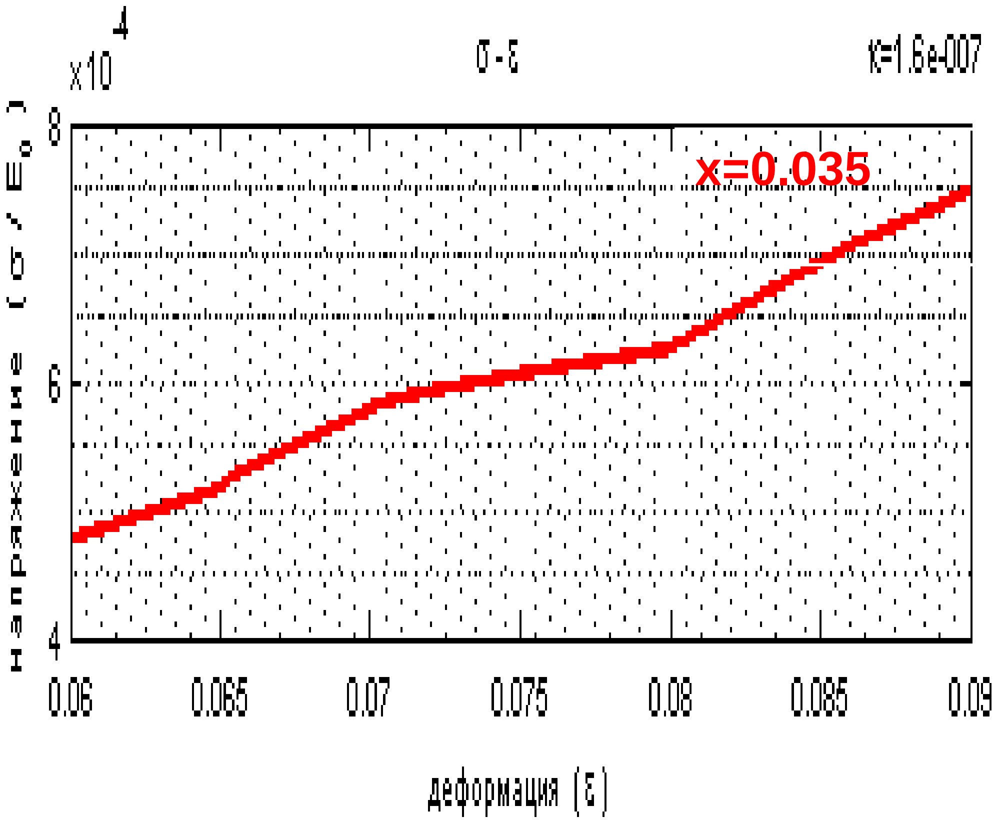
6

$$\sigma_1(0,t) = t \cdot a,$$

$$E_0 = E_0(\varepsilon), \qquad n^+(x,0) = n_0, \quad n^-(x,0) = 0$$

$$\frac{\partial E_0}{\partial \varepsilon}(\varepsilon_1) = 0, \quad \frac{\partial^2 E_0}{\partial \varepsilon^2}(\varepsilon_1) < 0 \qquad \alpha = 0, \quad \beta = const \cdot \sqrt{\varepsilon - \varepsilon_1}$$





Thank you for attention!